range of grain sizes and the other containing particles of a uniform size, are both tested (in air and vacuum) at the same unit weight (used as the criterion of denseness in these experiments), the well-graded material will be in a relatively looser state than the uniform soil, and greater depths of penetration will be recorded in it. Thus it is seen that the distribution of grain sizes also affects the results.

Further experiments are required to elucidate the effect of vacuum level and duration on the effective coefficient of friction of soils of different grain sizes.

Conclusions

In summary, it may be said that, on a qualitative theoretical basis, other factors being equal, dynamic penetration of freely falling probes into a densely packed cohesionless granular mass in air will be very much less than into a loosely packed granular material in air. Penetration into a loosely packed granular material in vacuum will be less than into a loosely packed granular material in air, and penetration into a densely packed mass in vacuum will be greater than into a densely packed mass in air. The results of the experiments tend to confirm these conclusions and other tentative qualitative deductions on the frictional behavior of soils exposed to vacuum conditions.

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Analysis of Foldability in Expandable Structures

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Outlines for a general theory of large deformations, including folding of arbitrary inextensible membranes, are presented. The approach to the problem uses isometric mapping techniques complemented by the additional topological constraints of the folding problem in real membrane structures. The theory is applied to an inextensible membrane in the form of a torus. Rigorous solutions are found for a particular class of deformations. Theoretical results are verified qualitatively by realization of predicted folding patterns on two torus models.

Nomenclature

E,F,G = Gaussian coefficients of the first fundamental form
R = radius of circumferential center line of torus
S = surface
$\mathbf{X}$ = radius vector
f,g,h,k = auxiliary functions of (u,v)
n = integer
p = deformation parameter
u,v = curvilinear surface coordinates
x,y,z = Cartesian coordinates
λ = direction of propagation on surface
ρ = radius of meridional circular torus section

Superscripts and subscripts

$*$ = function of the deformed surface
(u), etc. = variable of a function
u, etc. = partial derivative of a function with respect to the variable u

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I. Introduction

EXPANDABLE pneumatic structures, i.e., structures that can be packaged into small volumes and erected by inflation into relatively rigid devices, have been considered for a number of space missions. Of particular interest is the design of expandable large-sized manned orbital space laboratories in the form of a modified torus,[1] either partially or fully constructed from flexible materials.

Other examples where expandable structures can find applications are the large surface required for reflectors of electromagnetic radiation (Echo satellite), collectors for solar energy, and expanding and retracting devices for manipulation of instruments during flight, re-entry, or for operations after landing on foreign celestial bodies.

In many of these applications, the operating pressure and/or the size of the expandable structure is such that considerable structural forces arise from pressurization. This requires a wall construction that is strong and, as a consequence, stiff, at least in directions tangential to the surface. Thus, although optimum design and materials selection may result in a thin-walled strong shell that retains sufficient bending compliance to allow relatively sharp bending radii, these designs exhibit normally sufficient membrane stiffness to limit the membrane strains to small values. As a limiting

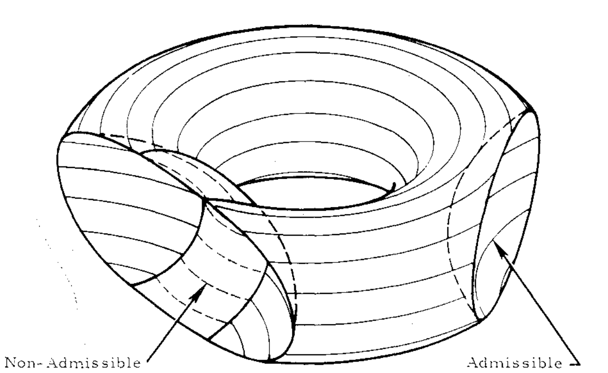

Fig. 1 Isometric deformation of torus membrane by reflection on intersecting planes

case, such structures can be considered as inextensible but completely flexible membranes.

A general theory of "momentless" (i.e., flexible) shells, with a detailed discussion of their inextensible deformation characteristics, is presented, for instance, in Ref. 2. The discussion in Ref. 2 is limited, however, to shells of revolution and concerns itself primarily with infinitesimally small deformations and finite bending radii in the normal sense of the theory of elastic structures. For the purpose of an analytical treatment of the packaging and folding problem of expandable structures, a more general theory is required. Such a theory can be developed based upon isometric mapping techniques.[3–5]

Other applications of the approach described here have come to the author's attention after the original preparation of this report. Hemp[7] has used differential geometrical relations for a very general treatment of shell deformations. Williams[8] uses general surface coordinates for studies of free surface conditions in liquids, in which case the tension rather than the metric of the surface remains invariant.

II. General Criteria for the Deformation of Inextensible Membranes

Consider a thin-walled structural shell. Its shape can be described by a neutral surface S located between the two faces of the shell. Assume that the neutral surface admits no membrane strains in tangential direction and that the shell is completely compliant in bending. Such a structural shape will be described as an inextensible membrane.

Let the neutral surface S be deformed continuously into a consecutive set of new surfaces $S^*(p)$, where p is a continuously varying parameter. For the corresponding inextensional membranes to be deformable into the consecutive shapes described by the parametric set of surfaces S^*, the following conditions need to be satisfied:

1) All surfaces of the set S^* must be isometric with S; i.e., the transformation $S \rightarrow S^*$ must retain all lengths (and, consequently, all angles) on the entire surface. Isometry of transformation satisfies the condition of zero membrane strain required by inextensible membranes.

2) In the domains where the original surface S is continuous, the surfaces of the set S^* also must be continuous. It will not be required, however, that the derivatives of the surfaces S^* be continuous at all points. Thus, the deformation may involve ridges and/or folds along certain lines that may be either fixed on the surface or traveling over the surface with a variation of the deformation parameter p. The admission of slope discontinuities for the deformation of membranes constitutes a departure from the usual conventions of deformations in thin shells. For instance, closed analytical surfaces of continuously positive curvature (egg-surfaces) normally are considered as rigid.[2, 4] This is true only if deformations involving slope discontinuities are excluded.

3) The topological characteristics of the surfaces S^* must be equal to the topological characteristics of the original surface S. This refers particularly to the surface connectivity (genus) and surface orientation (insides of closed surfaces must remain inside). The condition of invariant connectivity excludes, for instance, the case of mapping a closed, periodic surface upon an infinitely extended open surface. An example of this is given in the mapping of a torus upon a corrugated tube, discussed in Sec. IV. The topological condition of surface orientation must be applied to exclude those deformations which, although isometric and continuous, would require the membrane to change sides by mutual permeation.

An example of admissible and inadmissible isometric deformations generated by reflection of the surface on intersecting planes and involving ridge formation is shown in Fig. 1.

III. Basic Equations for Isometric Deformation of Surfaces

Let the inextensible membrane in consideration be represented by its neutral surface S. Its analytical expression may be given by the vector $\mathbf{X}_{(u,v)}$ extending from an origin O to a point P on the surface and referred to the three-dimensional Euclidian system of coordinates (x,y,z) as shown in Fig. 2.

The parameters u and v describe a parameter net of curvilinear coordinates u = const and v = const on the surface S. The vector $\mathbf{X}_{(u,v)}$ can be written in terms of its components as follows:

$$\mathbf{X}_{(u,v)} = \begin{pmatrix} x_{(u,v)} \\ y_{(u,v)} \\ z_{(u,v)} \end{pmatrix} \quad (1)$$

The "infinitesimal" vector $d\mathbf{X}$ from the point $P_{(u,v)}$ to the point $Q_{(u+du,v+dv)}$ is given by the components

$$d\mathbf{X} = \begin{pmatrix} x_u\,du + x_v\,dv \\ y_u\,du + y_v\,dv \\ z_u\,du + z_v\,dv \end{pmatrix} \quad (2)$$

where the subscripts refer to the partial derivatives: $x_u = \partial x/\partial u$, etc.

The absolute value of $d\mathbf{X}$ is equal to the length of the line element ds of the surface. The square of the differential length, ds^2, can be obtained by scalar multiplication of $d\mathbf{X}$ with itself:

$$(ds)^2 = (d\mathbf{X}\,d\mathbf{X}) = E_{(u,v)}\,du^2 + 2F_{(u,v)}dudv + G_{(u,v)}dv^2 \quad (3)$$

This is the "first fundamental form" of the surface S with the Gaussian fundamental functions of u and v:

$$\begin{aligned} E_{(u,v)} &= (x_u)^2 + (y_u)^2 + (z_u)^2 \\ F_{(u,v)} &= x_u x_v + y_u y_v + z_u z_v \\ G_{(u,v)} &= (x_v)^2 + (y_v)^2 + (z_v)^2 \end{aligned} \quad (4)$$

Consider now a second surface S^* that is represented by the vector $\mathbf{X}^*(u,v)$ with the coordinates $x^*_{(u,v)}$, $y^*_{(u,v)}$, $z^*_{(u,v)}$ referred to the same parameters u,v as $\mathbf{X}_{(u,v)}$. The two surfaces S and S^* are called locally isometrical if, in the points u,v on S, the differential length ds is equal to the differential length ds^* in the corresponding point u,v on S^*. This means that for arbitrary directions of propagation $\lambda = du/dv$ the equation

$$\left(\frac{ds}{ds^*}\right)^2 = \frac{Edu^2 + 2Fdudv + Gdv^2}{E^*\,du^2 + 2F^*dudv + G^*\,dv^2} = \frac{E\lambda^2 + 2F\lambda + G}{E^*\lambda^2 + 2F^*\lambda + G^*} = 1 \quad (5)$$

must be satisfied. Here $E^*_{(u,v)}$, $F^*_{(u,v)}$, $G^*_{(u,v)}$ are the Gaussian fundamental quantities of S^* referred to the same curvilinear surface coordinates u,v to which S is referred. The two surfaces S and S^* are entirely isometrical if Eq. (5) holds for all points (u,v) and for arbitrary directions λ. This is possible only if the following identities hold:

$$E = E^* \qquad F = F^* \qquad G = G^* \tag{6}$$

Let the surface S^* be represented by

$$\mathbf{X} = \begin{pmatrix} x^*_{(u,v)} \\ y^*_{(u,v)} \\ z^*_{(u,v)} \end{pmatrix} \tag{7}$$

The necessary and sufficient condition that S^* be isometrical to S is that the components x^*, y^*, z^* satisfy the following system of partial differential equations:

$$\begin{aligned} (x_u^*)^2 + (y_u^*)^2 + (z_u^*)^2 &= E \\ x_u^* x_v^* + y_u^* y_v^* + z_u^* z_v^* &= F \\ (x_v^*)^2 + (y_v^*)^2 + (z_v^*)^2 &= G \end{aligned} \tag{8}$$

where E, F, G are the Gaussian fundamental quantities of the original surface S.

The entirety of surfaces that are isometrical to the given surface S is obtained from the entirety of solutions x^*, y^*, z^* of the system (8).

Trivial solutions of (8) can be found by rigid body displacements:

$$\begin{aligned} x^* &= x + c_1 p \\ y^* &= y + c_2 p \\ z^* &= z + c_3 p \end{aligned} \tag{9a}$$

where c_1, c_2, c_3 are arbitrary constants and p is the continuously varying deformation parameter. Another class of isometric deformations is obtained by intersecting the surface by a plane and reflecting the portion of the surface on one side of the plane upon the other side, such as shown in Fig. 1. For instance, if the reflecting plane is parallel to the xy plane and is described by $z = p$, then the coordinates of the deformed surface are

$$\begin{aligned} x^* &= x \\ y^* &= y \\ z^* &= z \qquad & z < p \\ z^* &= 2p - z \qquad & z > p \end{aligned} \tag{9b}$$

This deformation generates normally a ridge along the line of intersection traveling on the surface with a change of location p of the reflecting plane.

Since the system of Eq. (8) is nonlinear in the derivatives of its functions, it will be difficult to find general solutions. In specific cases, it may be convenient to transform (8) into a linear system by the following substitution:

$$\begin{aligned} x_u^* &= E^{1/2} \cos f \cos g \\ y_u^* &= E^{1/2} \cos f \sin g \\ z_u^* &= E^{1/2} \sin f \end{aligned} \tag{10a}$$

$$\begin{aligned} x_v^* &= G^{1/2} \cosh \cos k \\ y_v^* &= G^{1/2} \cosh \sin k \\ z_v^* &= G^{1/2} \sinh \end{aligned} \tag{10b}$$

where $f_{(u,v)}$, $g_{(u,v)}$, $h_{(u,v)}$, $k_{(u,v)}$ are four auxiliary functions of u and v.

The first and third conditions of (8) are satisfied implicitly by (10a,b). The second condition in (8) yields the algebraic relation

$$\cos f \cosh \cos(g - k) + \sin f \sinh = F/(EG)^{1/2} \tag{11}$$

The integrability conditions for twice-differentiable domains

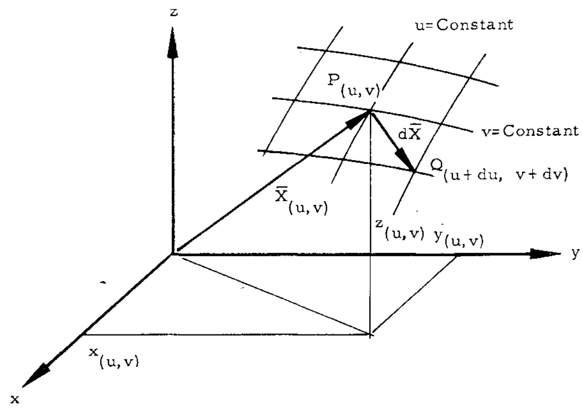

Fig. 2 Coordinate system for general surface

of the surface (i.e., domains excluding slope discontinuities) require

$$x_{uv}^* = x_{vu}^* \qquad y_{uv}^* = y_{vu}^* \qquad z_{uv}^* = z_{vu}^* \tag{12}$$

Differentiating (10a) and (10b) and substituting into (12), one obtains

$$\begin{aligned} (\partial/\partial v)(E^{1/2} \cos f \cos g) &= (\partial/\partial u)(G^{1/2} \cosh \cos k) \\ (\partial/\partial v)(E^{1/2} \cos f \sin g) &= (\partial/\partial u)(G^{1/2} \cosh \sin k) \\ (\partial/\partial v)(E^{1/2} \sin f) &= (\partial/\partial u)(G^{1/2} \sinh) \end{aligned} \tag{13}$$

The system (13) constitutes three simultaneous differential equations for the four functions f, g, h, k of u and v, which are, as an additional condition, related by the algebraic equation (11). These four equations are equivalent to the system (8) and may in specific cases be more convenient for the purpose of finding nontrivial, twice-differentiable isometric deformations.

A general solution will not be attempted here. Instead, the specific case of an inextensible torus membrane will be investigated.

IV. Isometric Deformation of a Circular Torus

A class of deformations for a torus, as shown in Fig. 3, can be obtained explicitly by integration of Eqs. (11) and (13). For the coordinate system shown, the radius vector to a point (u,v) on the torus is given by

$$\mathbf{X} = \begin{pmatrix} x_{(u,v)} \\ y_{(u,v)} \\ z_{(u,v)} \end{pmatrix} = \begin{pmatrix} (R + \rho \cos u) \cos v \\ (R + \rho \cos u) \sin v \\ \rho \sin u \end{pmatrix} \tag{14}$$

where R is the distance from the origin O to the center line of

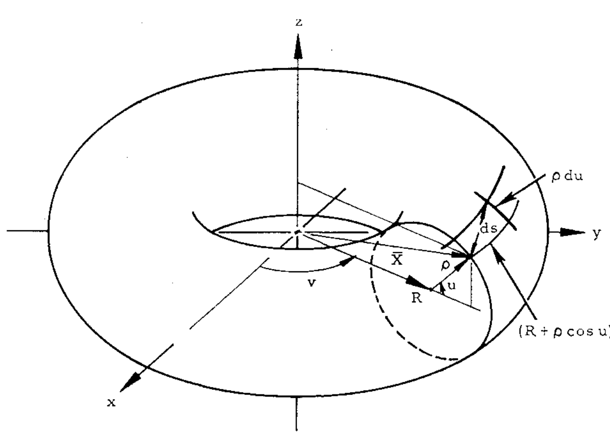

Fig. 3 Torus coordinates

the torus, and ρ is the radius of the meridional circle which generates the torus by revolution about the z axis.

The curvilinear coordinates u = const and v = const, in this case, represent parallel circles and meridians, respectively; u is the angle between the radius ρ and the x-y plane, and v is the central angle between the plane containing the meridian v = const and the x-z plane. The coordinates u and v are equivalent to the latitude and longitude angles conventionally used as spherical coordinates.

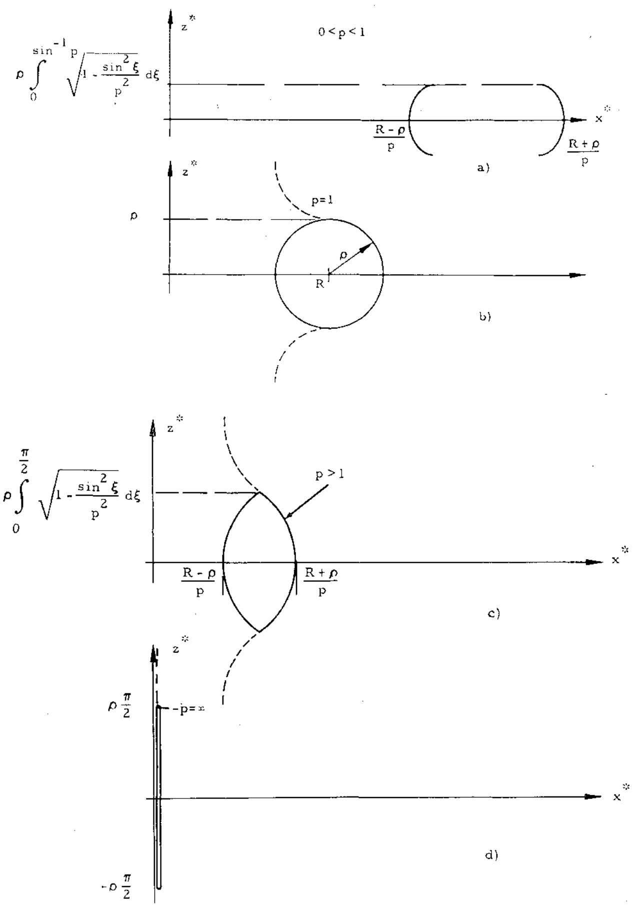

Fig. 4 Solutions for isometric torus sections according to Eq. (20)

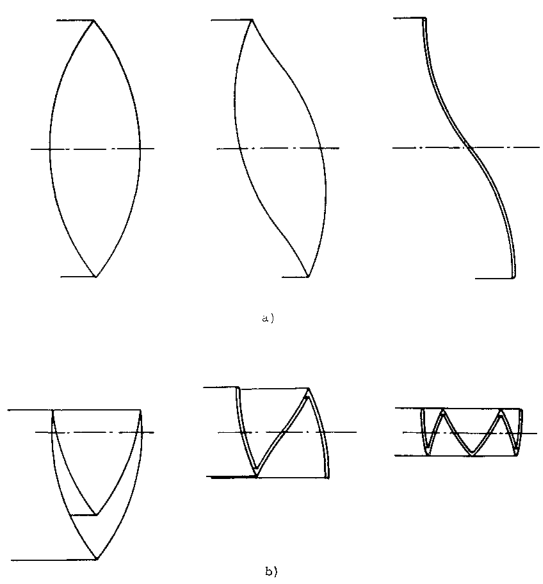

Fig. 5 Closed isometric, meridional torus cross sections obtained by reflection

By inspection of the coordinate geometry shown in Fig. 3, the line element of the torus is

$$ds^2 = \rho^2 du^2 + (R + \rho \cos u)^2 \, dv^2 \tag{15}$$

and the Gaussian fundamental quantities by comparing (15) with (3) become

$$E = \rho^2 \qquad F = 0 \qquad G = (R + \rho \cos u)^2 \tag{16}$$

Solutions for isometric deformations will now be restricted to those where parallel circles remain curves in parallel planes ($z_v{}^* = 0$).

With the expressions in (16), a set of solutions of (11) and (13) then can be given by

$$\begin{aligned} f &= \cos^{-1}(\sin u/p) \\ g &= -\pi + pv \\ h &= 0 \\ k &= pv + (\pi/2) \end{aligned} \tag{17}$$

Inserting these solutions into Eqs. (10a) and (10b) yields

$$\begin{aligned} x_u{}^* &= -(\rho/p) \sin u \cos pv \\ y_u{}^* &= -(\rho/p) \sin u \sin pv \\ z_u{}^* &= \rho[1 - (1/p^2) \sin^2 u]^{1/2} \end{aligned} \tag{18a}$$

and

$$\begin{aligned} x_v{}^* &= -(R + \rho \cos u) \sin pv \\ y_v{}^* &= (R + \rho \cos u) \cos pv \\ z_v{}^* &= 0 \end{aligned} \tag{18b}$$

From these equations, the components of $\mathbf{X}^*$ describing the surface S^* can be obtained by quadrature:

$$\begin{aligned} x^* &= (1/p)(R + \rho \cos u) \cos pv - c_1 \\ y^* &= (1/p)(R + \rho \cos u) \sin pv - c_2 \\ z^* &= \rho \int_{u_0}^{u} \left[1 - \frac{1}{p^2} \sin^2 \xi \right]^{1/2} d\xi - c_3 \end{aligned} \tag{19}$$

The three integration constants c_1, c_2, c_3 represent a rigid body translation that can be disregarded for further discussion.

In this case, the surface S^* is generated by revolution of a meridional curve defined in the x-z plane by the parametric relation

$$\begin{aligned} x_{(u)}{}^* &= (1/p)(R + \rho \cos u) \\ z_{(u)}{}^* &= \rho \int_{u_0}^{u} \left[1 - \frac{1}{p^2} \sin^2 \xi \right]^{1/2} d\xi \end{aligned} \tag{20}$$

The integral expression of the second equation in (20) represents an elliptical integral of the second kind. Values for this integral, tabulated in Ref. 6, have been used for the construction of the meridional curves discussed in Sec. V.

V. Discussion of Results

Solutions for the meridional shapes according to Eq. (20) are shown in Fig. 4 for selected parameters p. If p is any value between zero and one, the curve consists of segments of real branches (Fig. 4a). The openings between these branches correspond to parameter values $u > \sin^{-1} p$ (i.e., to those values of u for which the radicand $[1 - (1/p^2) \sin^2 u]$ is negative). These solutions cannot satisfy the topological restraints for a complete torus surface and therefore will not be considered further.

It will be observed that the meridional curves described by (20) even for $p \geq 1$ are not necessarily closed; thus the conditions of equal topological connectivity between S and S^* are not satisfied a priori. Closed meridians can be obtained by axial folding, i.e., by reflection on a plane z = const through the parallel circles $u = \pm(\pi/2)$. The result is a lenticular section with two ridges, as shown in Fig. 4c. This reflection can be expressed mathematically by the convention that the square root under the integral in (20) be taken positive for $-\pi/2 < u < \pi/2$ and negative for $\pi/2 < u < 3\pi/2$.

With the convention of simple reflection at $u = \pm(\pi/2)$, closed curves are obtained for all $p \geq 1$. For $p = 1$, a circle is obtained which generates exactly the original torus (Fig. 4b). As p approaches infinity, the meridional curve degenerates into a line covering twice the z axis from $-\rho(\pi/2)$ to $+\rho(\pi/2)$ (Fig. 4d).

A set of more general closed meridional sections can be obtained by reflection on planes through u = const and $u + \pi$ = const, as shown for the case $p = 2$ in Fig. 5a. Further shapes, particularly shapes of vanishing cross-sectional area, may be obtained by subsequent reflections on other planes z = const, as shown in Fig. 5b.

A similar situation exists with respect to the circumferential coordinate v. Topological connectivity of the surface in circumferential direction requires that the surface S^* be periodic in v with the period 2π. This can be accomplished, for instance, by a circumferential folding technique as follows.

Consider n equal segments of the deformed torus where the end meridians of each segment enclose a central angle of $2\pi(p/n)$. Each segment now can be reflected on a vertical plane bounded by the z axis, intersecting the segment at an angle $[(p + 1)/n]\pi$. By this reflection, the segment will be folded into itself, and the increment in central angle between end meridians becomes $2\pi/n$. By joining all n segments, the topological periodicity condition that the end of the last segment $v = 2\pi$ coincide with $v = 0$ is satisfied. Figure 6 shows a circumferential folding schematic for $p = 3$, $n = 2$.

By this method, certain domains of the deformed surface are covered by the membrane in multiple layers. The minimum number of layers is three for $1 < p < 3$. For $p = 3$, the whole torus domain is covered triply. Further increase of $3 < p < 5$ will require quintuple coverage of certain domains up to $p = 5$, etc.

Finally, it should be remarked that the necessity for circumferential folding disappears if the torus can be cut along any meridian (torus-segment). Such a structure may be folded into a tight scroll of vanishing enclosed volume and frontal area.

VI. Experimental Realization

Qualitative realization of the theoretical data presented has been obtained by experimentation with two torus models.

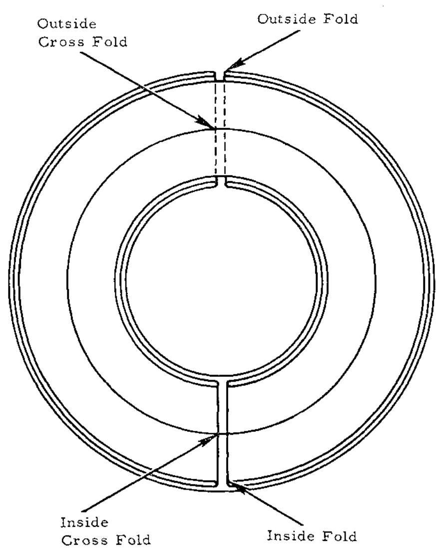

Fig. 6 Circumferential folding schematic of torus for $n = 2$, $p = 3$

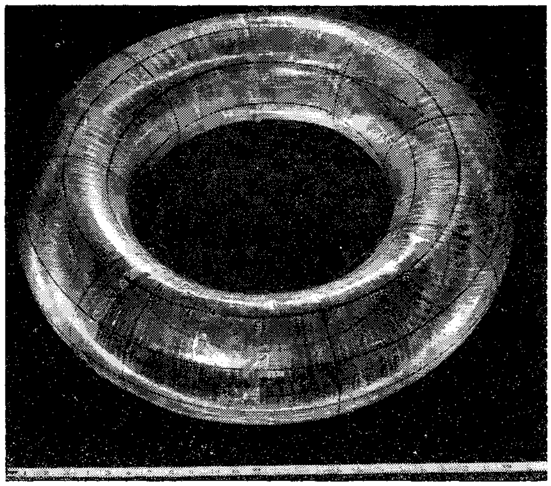

Fig. 7 Full torus, expanded

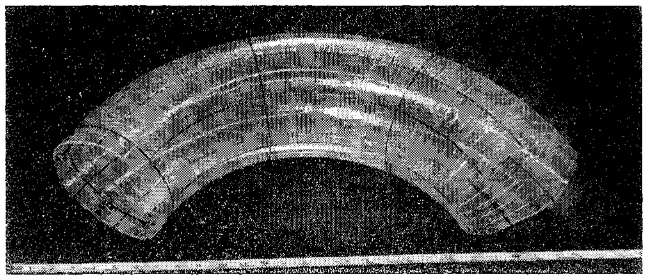

Fig. 8 Quarter torus, expanded

For this purpose, a full and a quarter torus have been fabricated with the following overall dimensions: $R = 19.5$ in. and $\rho = 3.5$ in.

The method of fabrication consists of winding two overlapping layers of 2.5-mil thickness adhesive coated tape (trademark "Scotchtape") on an inflatable mandrel made from a standard size 670-15 automotive inner tube. The tape is applied in such a manner that the adhesive coated sides of the two layers are in mutual contact. This process results in an average wall thickness of 7 mil. After completion of the winding process, the rubber tube is removed through a slit, and the slit is repaired for the closed torus by an overlay of tape. The models fabricated in this fashion approximate closely the idealized conditions of inextensible membranes.

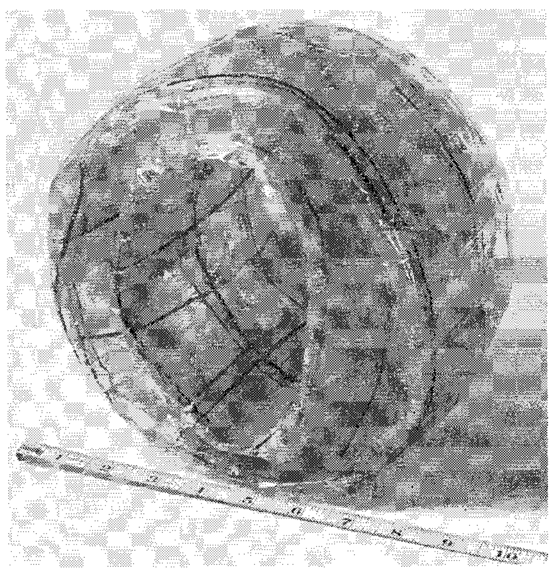

Fig. 9 Full torus, folded ($p = 3$, $n = 2$)

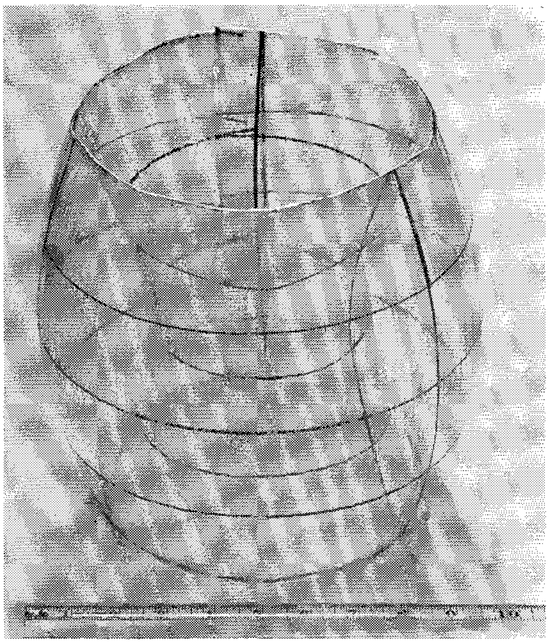

Fig. 10 Quarter torus, folded ($p = 4$)

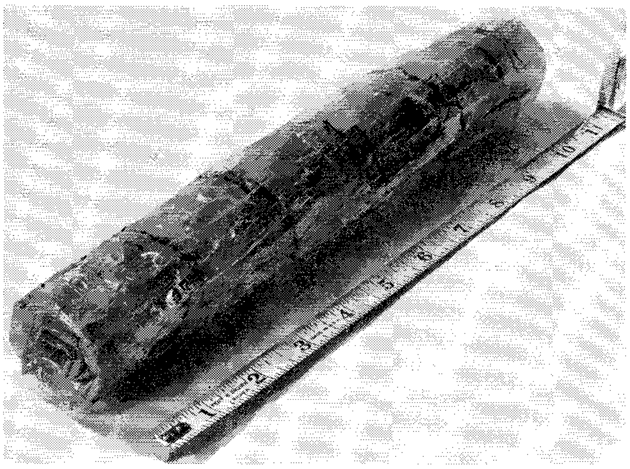

Fig. 11 Quarter torus, folded ($p \rightarrow \infty$)

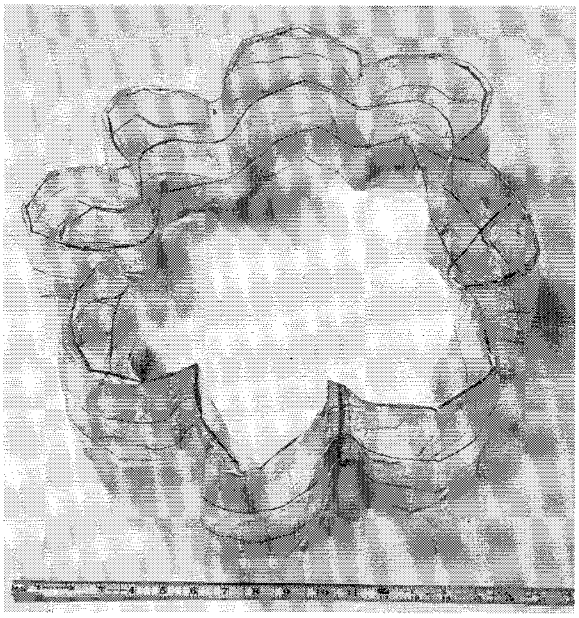

Fig. 12 Full torus, folded (epicycloid)

The two torus models are shown in Figs. 7 and 8 in their expanded condition. Figure 9 shows the folded shape of the complete torus with circumferential and axial folding according to the folding schematic shown in Figs. 5b and 6. Figure 10 shows the quarter torus segment folded into a closed shell ($p = 4$) exhibiting the predicted lenticular meridional shape. Figure 11 shows the torus segment in a tight scroll according to Fig. 4d.

An interesting variant of folding deformation deviating from rotational symmetry is shown in Fig. 12. This shape involves deformation of the original parallel circles u = const into leafed curves resembling epicycloids. The reverse fold required at the cusps between leafs is topologically possible since the cross section at these meridians degenerates into a double line at these locations. Although, in principle, the leafed shape is possible for a completely closed torus, attempts to produce this pattern from the original full torus were not successful, indicating that no continuous isometric and topologically invariant set S^* exists between the leafed "epicycloid" shape and the original complete circular torus.

VII. Concluding Remarks

It is clear that, for instance, the circumferential folding technique shown in Fig. 6 in its pure form is possible only for infinitely thin membranes. For practical structural shells of finite thickness, such a problem can be overcome, for instance, by a periodic variation of the torus cross section, allowing finite spacing of the concentric layers. Furthermore, axial folding involving concave folds such as shown in Figs. 5b and 9 can be used to reduce the difficulty in circumferential folding.

A second, possibly more serious practical difficulty, is the presence of stationary and traveling cross-folds (i.e., folds crossing ridges), as indicated in Fig. 6. Practical implementation may require specific provisions in the wall design allowing for finite membrane strains in the domains occupied by cross-folds. Other possibilities may be provided by different folding patterns, such as those of the type shown in Fig. 12, which may eliminate traveling cross-folds.

Further study should be directed towards isometric deformations that do not necessarily retain rotational symmetry. Also of interest will be the expansion of the general theory to shells that admit small but finite membrane strains. Such an expansion will be particularly useful for a study of local-fold and cross-fold areas.

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